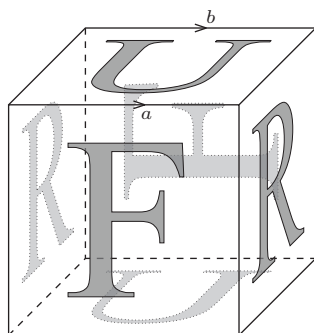


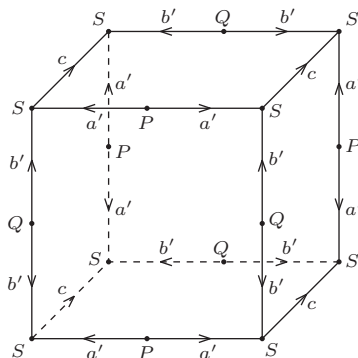
## Addendum to Section 4.3 of *Topology Now!*

There is one situation in which gluing orange slices together around an edge does not result in a ball as is mentioned at the top of page 138. The following exercise illustrates this case and shows how to deal with it.

1. Consider a solid cube with front and back faces identified with a quarter twist and top and bottom faces identified with a reflection as indicated in the figure below.



- a. Show that the edge labeled  $a$  is identified with three other edges, and then is identified with itself with its orientation reversed. Show edge  $b$  suffers the same fate.
- b. Argue that the four orange-slice pieces of the neighborhood of the point  $P$  at the center of edge  $a$  are glued together to form a solid whose boundary is a projective plane rather than a sphere. Show that the same behavior occurs around the point  $Q$  at the center of edge  $b$ . It follows that this pseudo-manifold is not a manifold.
- c. Here is a way to reconcile this situation with the situation analyzed in the text where edges are glued with consistent orientations. Add two new vertices,  $P$  in the center of edge  $a$  and  $Q$  in the center of edge  $b$ . Let  $a'$  be half of  $a$ , and let  $b'$  be half of  $b$ . Check that edges and vertices are glued as shown below.



- d. Check that the Euler characteristic of this pseudo-manifold is nonzero.
- e. Check that the vertex  $S$  has a neighborhood bounded by the connected sum of two projective planes.
- f. Argue that all interior points of edges  $a'$ ,  $b'$ , and  $c$  have neighborhoods bounded by spheres. Hence, vertices  $P$ ,  $Q$ , and  $S$  are the only points that do not have neighborhoods that are solid balls.