Additional Exercises for Chapter 1 of *Topology Now!*

1. a. Set up a bijection between the pairs of handle positions on a two-handled faucet and the ordered pairs where the first coordinate is the temperature and the second coordinate is the flow rate.

   b. What can you say about the correspondence between the handle positions on a single-handled faucet and the ordered pairs of temperature and flow rate?

2. Suppose we defined two sets X and Y to be equivalent if and only if there is a surjection $f : X \to Y$. Is this an equivalence relation? Justify your answer.

3. Below are the lowercase letters of the Greek alphabet, beloved of mathematicians. Think of these letters as made of one-dimensional arcs that include their endpoints.

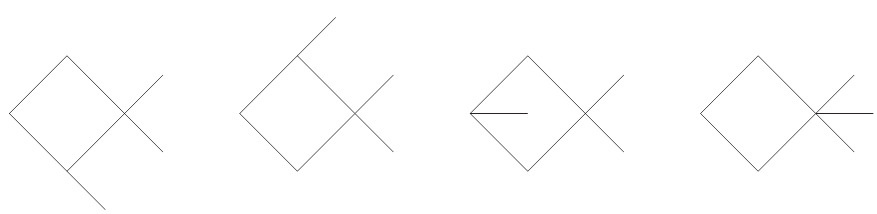
   α β γ δ ε ζ η θ ι κ λ μ

   ν ξ ο π ρ σ τ υ φ χ ψ ω

   a. Partition these letters into homeomprphism classes.

   b. Use topological invariants to demonstrate that α, β, θ, and ξ are all in different homeomorphism classes.

4. Which of the four objects below are ambient isotopic in $\mathbb{R}^2$? If you claim that two figures are isotopic, show several stages of an ambient isotopy between them. If you claim that two figures are not isotopic, give a convincing reason that they are not.



5. Define two sets to be **point-related** if and only if we can remove a point from each of them so that the remaining sets are homeomorphic. That is, X and Y are point-related if and only if there is a point $p \in X$ and a point $q \in Y$ such that $X - \{p\}$ is homeomorphic to $Y - \{q\}$.

   a. Is the circle $S^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ point-related to the half-open interval $[0, 1)$? Justify your answer.

   b. Is the interval $[0, 1)$ point-related to the set $[0, 1) \cup (1, 2) \cup \{3\}$? Explain.

   c. Is point-related an equivalence relation? Why or why not?