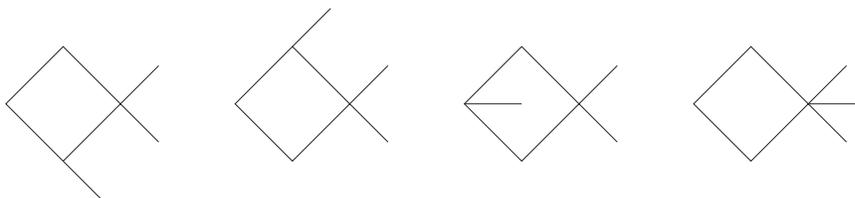


Additional Exercises for Chapter 1 of *Topology Now!*

1. a. Set up a bijection between the pairs of handle positions on a two-handled faucet and the ordered pairs where the first coordinate is the temperature and the second coordinate is the flow rate.
- b. What can you say about the correspondence between the handle positions on a single-handled faucet and the ordered pairs of temperature and flow rate?
2. Suppose we defined two sets X and Y to be equivalent if and only if there is a surjection $f : X \rightarrow Y$. Is this an equivalence relation? Justify your answer.
3. Below are the lowercase letters of the Greek alphabet, beloved of mathematicians. Think of these letters as made of one-dimensional arcs that include their endpoints.

$\alpha \quad \beta \quad \gamma \quad \delta \quad \varepsilon \quad \zeta \quad \eta \quad \theta \quad \iota \quad \kappa \quad \lambda \quad \mu$
 $\nu \quad \xi \quad \omicron \quad \pi \quad \rho \quad \sigma \quad \tau \quad \upsilon \quad \phi \quad \chi \quad \psi \quad \omega$

- a. Partition these letters into homeomorphism classes.
- b. Use topological invariants to demonstrate that α , β , θ , and ξ are all in different homeomorphism classes.
4. Which of the four objects below are ambient isotopic in \mathbb{R}^2 ? If you claim that two figures are isotopic, show several stages of an ambient isotopy between them. If you claim that two figures are not isotopic, give a convincing reason that they are not.



5. Define two sets to be **point-related** if and only if we can remove a point from each of them so that the remaining sets are homeomorphic. That is, X and Y are point-related if and only if there is a point $p \in X$ and a point $q \in Y$ such that $X - \{p\}$ is homeomorphic to $Y - \{q\}$.
 - a. Is the circle $S^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ point-related to the half-open interval $[0, 1)$? Justify your answer.
 - b. Is the interval $[0, 1)$ point-related to the set $[0, 1) \cup (1, 2) \cup \{3\}$? Explain.
 - c. Is point-related an equivalence relation? Why or why not?